Exploring Light Amplification by Stimulated Emission in Lasers ©

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Abstract

This document allows the students to use a simple approach in order to study the time evolution of the power output of a laser and its dependence upon the amplification coefficient, the length of the active medium, and the reflectivity of the cavity mirrors. The document starts with a problem on finding the laser power at a given moment of time for the different values of the mirror reflectivity. After solving this problem, the students are guided to predict the optimal value of the mirror reflectivity using simple calculus, to check their prediction using Mathcad, and to present a physical explanation of their findings. Then they proceed to use Mathcad to analyze the dependence of the output intensity on the amplification coefficient, the length of the cavity, and time passed since the laser has been switched on. Afterwards, students design a hypothetical laser and present their design strategies and the results in their report. Finally, we discuss the limitations of the simple model used, and outline the more rigorous approach to study the dynamics of laser generation. The material in this exercise can be used within a first course in Physical Chemistry where lasers and their applications in chemistry are introduced.

This document is supplemented by the separate Notes for the Instructor.

Goal

To demonstrate how light amplification in a laser depends upon the amplification of the active medium, its length, and the reflectivity of the cavity mirrors.

Objective

At the end of this exercise, students will be able to determine how each parameter associated with the power of the laser output affects that output.

Prerequisites

1) Knowledge of light absorption theory (stimulated and spontaneous emission, stimulated absorption).
2) Previous use of Mathcad.

As you should remember from your introductory Physical Chemistry course, the basis of the lasing action is stimulated emission of photons by the molecules in the active medium. Because of this amplification of light by the active medium, when the beam moves the distance L inside the active medium, its intensity increases from the initial value I_0 to

$$I = I_0 * \exp(a*L),$$

where a is called the amplification coefficient of the medium. In this document, we will use a very simple model in which a is treated as a constant. We will discuss the limitations of this model later.

We start with the following problem:

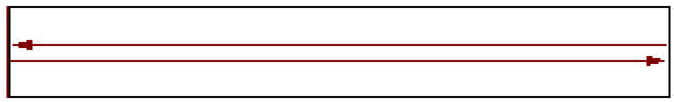
PROBLEM

At the moment t=0, the light intensity in the 10 cm-long cavity is 10^{-5} W. Estimate the intensity of the laser beam 100 ns later assuming that one of the mirrors of the laser cavity is 100% reflective, and the other one lets through (a) 10%, (b) 20% of the incident radiation. Suppose that the amplification coefficient, a, of the active medium (which fills the whole length of the cavity) is constant, $a = 1.0\ m^{-1}$.

If you are unsure how to approach the solution, consider the following hint:

Hint: **Track the fate of a part of the laser beam that makes one closed "cycle" inside the cavity.**

Hopefully, you have solved the problem; let's compare our solution path and results. Let's consider indeed what happens to a laser beam of intensity I_0 when it makes one "cycle" inside the cavity, as shown in the picture below. (Here the 100% reflective left mirror is shown by a bold line).



Let's call the initial light intensity near the left mirror I_0 (let's play with letters, not numbers for now; it's very easy to plug in the numbers whenever you wish working with Mathcad!!) . As you recall from the brief introduction above, when the beam moves the distance L inside the active medium to reach the right mirror, its intensity increases to

$$I = I_0 * \exp(a*L).$$

At the right mirror, p-th fraction of this intensity will be let through the mirror, while the remaining (1-p) -th part will be reflected back toward the left mirror. As it moves left, it will get amplified by the active medium, again by the factor exp(a*L). How do we find the resulting intensity I_1 at the end of the "cycle"? - We just multiply all the three factors together! Thus,

$$I_1 = I_0 * \exp(a*L) * (1-p) * \exp(a*L) =$$
$$= I_0 * (1-p) * \exp(2a*L) .$$

What happens after the end of this first cycle? - Light of intensity I_1 is reflected back to the right , and the whole cycle repeats again and again! Clearly, at the end of the SECOND cycle, the light intensity near the left mirror will be

$$I_2 = I_1 * \exp(a*L) * (1-p) * \exp(a*L) =$$
$$= I_1 * (1-p) * \exp(2a*L)=$$
$$= I_0 *[(1-p) * \exp(2a*L)]^2;$$

at the end of the THIRD cycle,

$$I_3 = I_2 * \exp(a*L) * (1-p) * \exp(a*L) =$$
$$= I_2 * (1-p) * \exp(2a*L)=$$
$$= I_0 *[(1-p) * \exp(2a*L)]^3$$

and so on. Thus, at the end of the n-th cycle,

$$I_n = I_{n-1} * \exp(a*L) * (1-p) * \exp(a*L) =$$
$$= I_{n-1} * (1-p) * \exp(2a*L)=$$
$$= I_0 *[(1-p) * \exp(2a*L)]^n.$$

The only thing remaining to be done is to calculate the number of cycles, n, the light goes through by the time t. Clearly, as light must pass the distance 2L for each cycle,

$$n=(ct)/2L,$$

where c is the speed of light.

Therefore, after time t the light intensity **inside** the cavity is going to be

$$I_n = I_0 *[(1-p) * \exp(2a*L)]^{ct/2L}.$$

The **output** laser intensity is, as we recall, p-th fraction of I_n, or

$$I_{las} = p* I_0 *[(1-p) * \exp(2a*L)]^{ct/2L}.$$

At last, we have the formula! Now let's play with numbers using Mathcad. Below we incert the numerical values of the parameters and define our function I(p). Now try to study it's behavior on your own!

$p := 0.0010, 0.0011 .. 0.21$

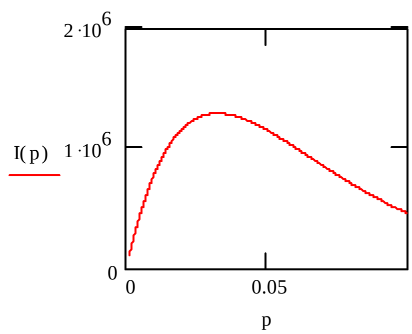
$a := 1 \cdot m^{-1}$

$c := 3 \cdot 10^{8} \cdot \frac{m}{sec}$ $Ii := 10^{-5} \cdot watt$

$L := .5 \cdot m$

$t := 10^{-7} \cdot sec$

$I(p) := p \cdot Ii \cdot ((1 - p) \cdot exp(2 \cdot a \cdot L))^{c \cdot \frac{t}{2 \cdot L}}$



EXERCISES:

1) Before you solved the problem, you may have thought qualitatively that the greater the fraction of the radiation you let through the mirror out from the cavity, the greater the output laser intensity should be (after all, if this fraction is 0%, the output laser power is certainly zero; then 10% must be better than 5%, 20% better than 10%, and so on). But is it true?! Given the values of L, a, and t, can you predict (without using the Mathcad) at what value of p is the laser output power reaching its maximum?

Hint: **find the derivative of I(p).**

2) Check your result using Mathcad.

3) Using physical arguments, explain why I(p) is initially increasing with increasing p but then starts to decrease?

Hint: **you might wish to think about the dependence of the output on the difference between the gain and losses in the cavity.**

4) Write a one-page essay with your answers to 1) - 3). Explain your conclusions.

5) In addition to I(p), study (using Mathcad) the dependence of the output intensity on
(a) the amplification coefficient of the active medium, a
(b) on the length of the cavity, L
(c) on the time passed, t (Consider both the case p=0.1 and p=0.2).

$p := 0.1$ [Type the needed values of p as you proceed with the analysis]

$a := .5 \cdot m^{-1}$ [Type the needed values of a as you proceed with the analysis]

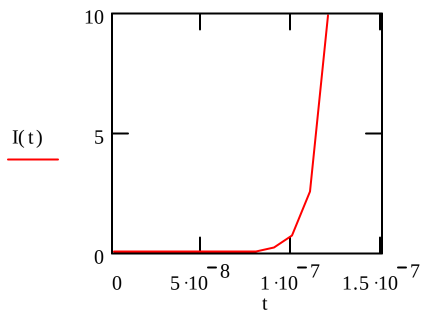
$Ii := 10^{-5} \cdot watt$

$c := 3 \cdot 10^{8} \cdot \frac{m}{sec}$

$L := 1 \cdot m$ [Type the needed values of L as you proceed with the analysis]

$t := 0 \cdot sec, .1 \cdot 10^{-7} \cdot sec .. 1.2 \cdot 10^{-7} \cdot sec$ [Type the needed values of t as you proceed with the analysis]

$$I(t) := p \cdot Ii \cdot ((1-p) \cdot \exp(2 \cdot a \cdot L))^{c \cdot \frac{t}{2 \cdot L}}$$



EXERCISES (continued):

6) In 5(c), when you consider times much larger than 100 ns, what is the trend for the output laser intensity? Do you think this is a true physical result or an aberration generated by the limitations of the model we used? Explain.

7) Explain your observations qualitatively. Add another page or two to your essay (4) reporting your findings on the questions 5)-7).

8) You are designing a laser to be used in "Star Wars". Suppose the length of the laser should not exceed 1 m to be mounted on the spaceship. What should be the minimal amplification of the active medium if $I_0=10^{-6}$ W and the output power of the laser should reach 1 watt in 1 microsecond? Present your design strategy and the results at the end of your essay.

Reflections:

As you could see, the power output of the laser depends drastically on the difference between the light amplification in the active medium and the losses (for example, through the semi-transparent mirror). Once amplification exceeds losses, the intensity of the output beam increases very rapidly. Do you recall how many times the value of I in the problem above has increased while p has decreased just from 0.2 to 0.1?! The steepness of the I(p)-dependency is one of the main reasons why it is so easy to manipulate the laser output.

Notice that in this module we have been using a very crude approximation regarding the amplification of the laser medium: we assumed that 'a' is just a constant. In reality, it depends upon many factors, the extent of population inversion among them. The approximation of constant a can be used at low light intensities; but as soon as those intensities become large, the stimulated emission leads to the decrease of population inversion and consequently to the decrease of the amplification coefficient. In order to accurately describe the dynamics of the laser generation, one needs to use a set of three coupled differential equations describing the time evolution of the electric field inside the cavity, polarization of the active medium, and the population inversion in it. For those of you who became seriously interested in the topic, I recommend, for example, the following book:

A.E. Siegman, "Lasers". University Science Books, Mill Valley, California, 1986, pp. 923-1128.